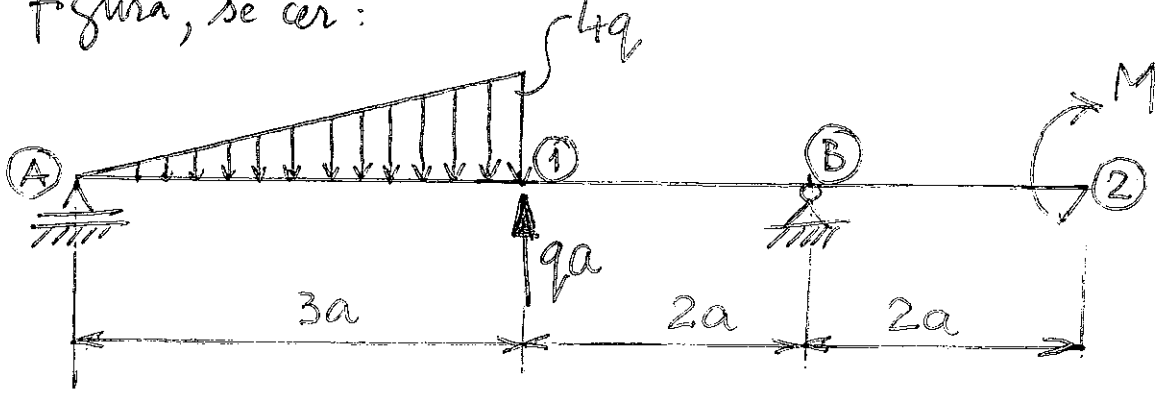
UNIVERSITATEA TEHNICĂ "GHEORGHE ASACHI" DIN IAȘI * FACULTATEA DE MECANICĂ
CONCURSUL STUDENȚESC DE REZISTENȚA MATERIALELOR – FAZA NAȚIONALĂ – 12 MAI 2017

SUBIECTELE PENTRU PROFILUL MECANIC

Problema 1

Pentru grinda cu forma, dimensiunile și secțiunea din figură, se cer:

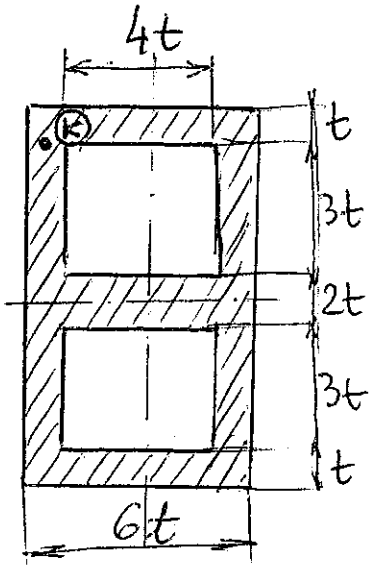


a) Valoarea momentului M astfel încât reacțiunile din punctele de reazem să fie egale;

b) Diagramele de eforturi (literal);

c) Să se dimensioneze secțiunea grinzii (t=?) dacă $\sigma_a = 150$ MPa, $a = 0,3$ m, $q = 10$ kN/m.

d) Tensiunile principale σ_1, σ_2 în punctul (K) din secțiunea (1)



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Problema 2

Bara de mare rigiditate BCDE din figură este susținută de două tije elastice 1 și 2. Tija 2 are secțiune neomogenă. Se cunosc:

$E_{OL} = 3E_{Al} = 21 \cdot 10^4$ MPa, $a = 0{,}5$ m, $F = 30$ kN,

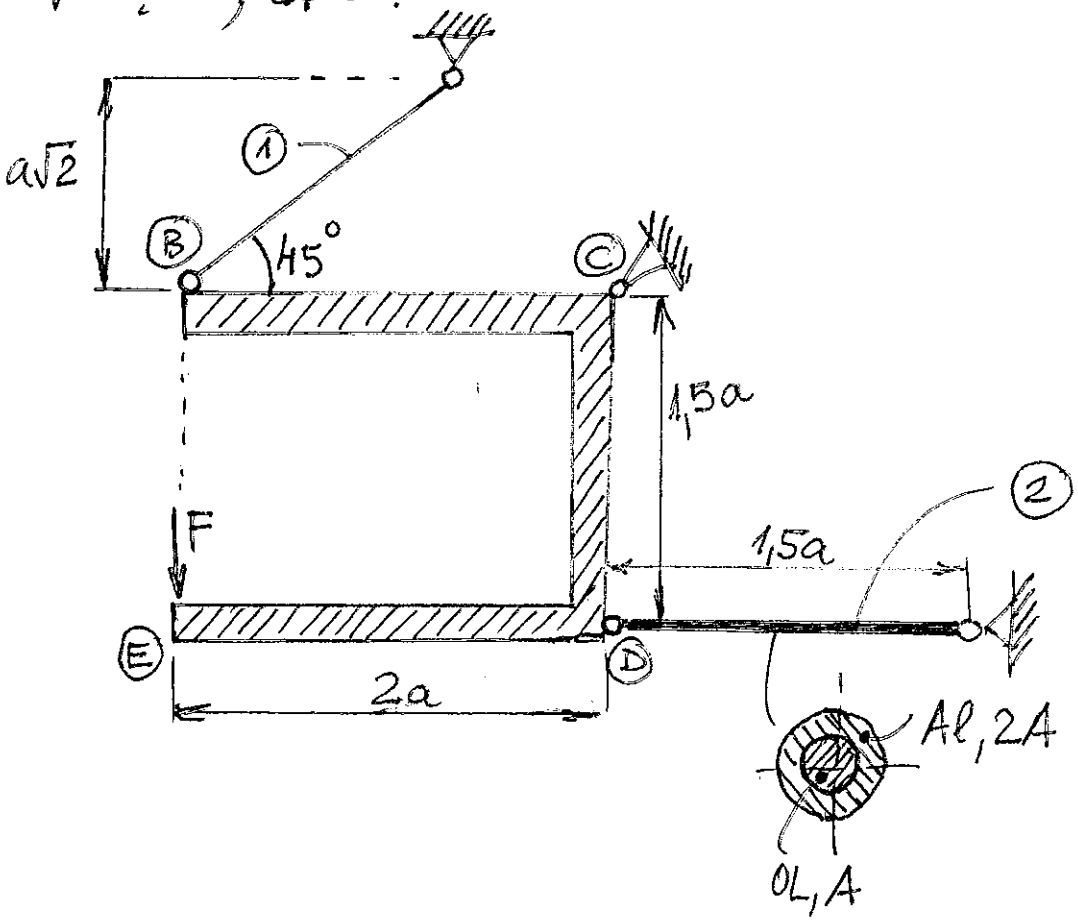
$\sigma_{a_{OL}} = 150$ MPa; $\sigma_{a_{Al}} = 100$ MPa.

Se cer:

a) Eforturile din cele două tije N_1^{OL}, N_2^{OL}, N_2^{Al};

b) Să se dimensioneze secțiunea tijelor $A_{nec} = ?$

c) Să se calculeze deplasarea punctului de aplicație al forței F, $\delta_F = ?$



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Problema 3

Grinda de secțiune dreptunghiulară $(b \times 2b)$, încastrată în secțiunea (A), are încărcările din figură:

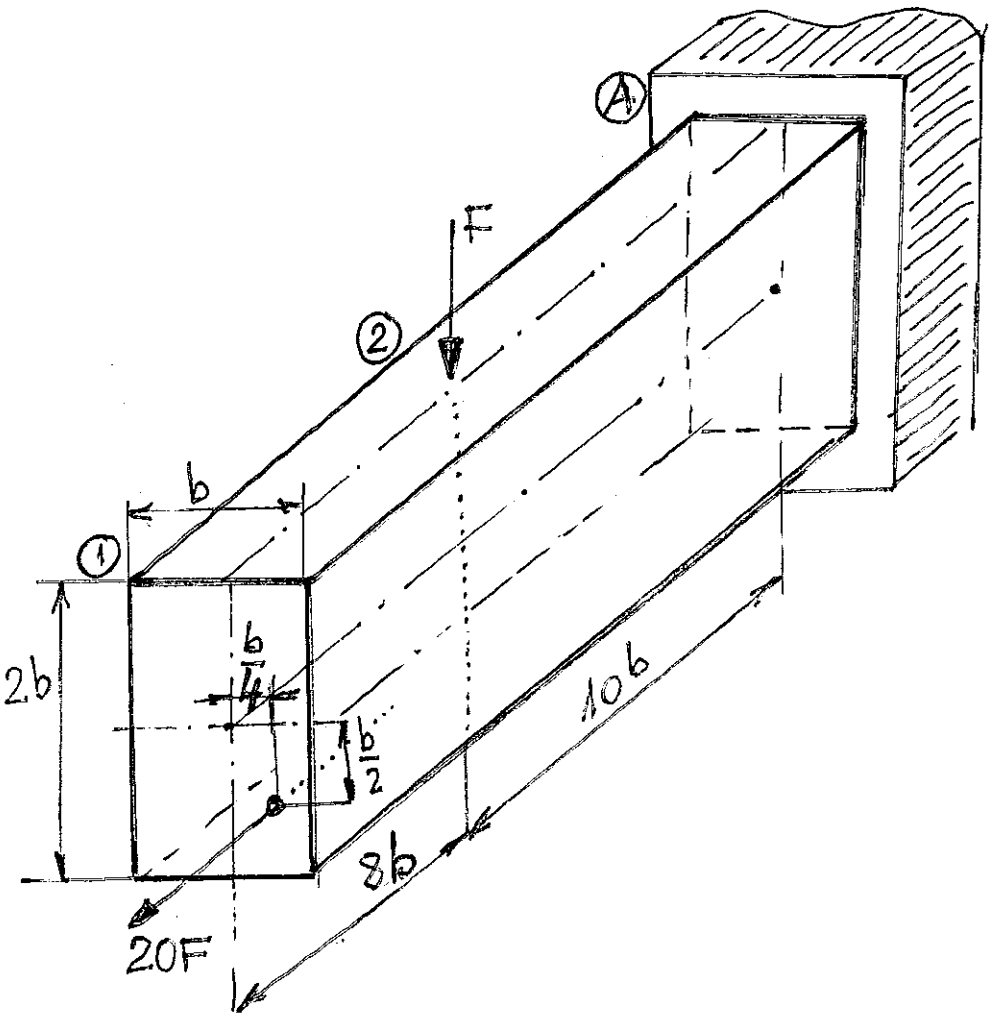
Se cer:

a) Diagramele de eforturi (N și M_i);

b) Ecuația axei neutre în secțiunea periculoasă;

c) Raportul $\left|\frac{\sigma_{max}}{\sigma_{min}}\right|$ în secțiunea periculoasă;

d) Deplasarea secțiunii de capăt (1) pe direcția forței axiale. Se cunosc: F, b, E.



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Problema 4

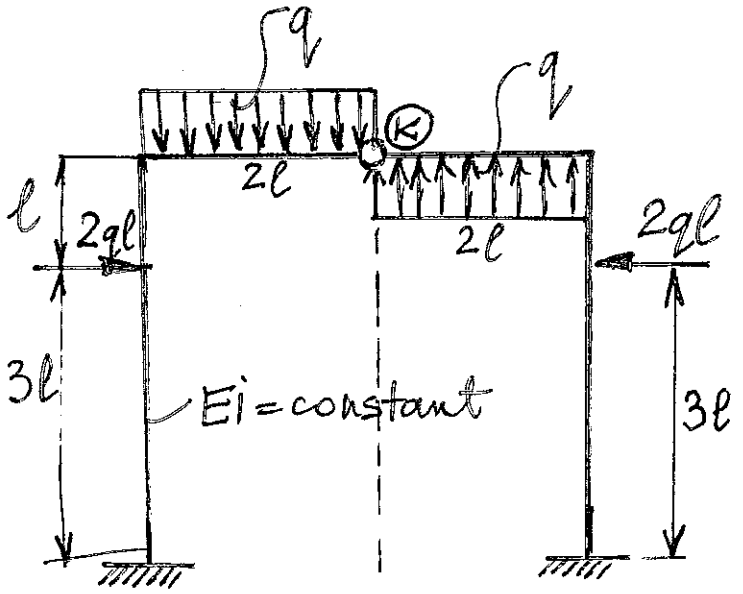
Cadrul din figură, de rigiditate constantă, are încărcările precizate.

Se cer:

a) Să se ridice nedeterminarea;

b) Să se traseze diagramele de eforturile N, T, M;

c) Să se calculeze deplasarea pe verticală în secțiunea K. Se cunosc: E, I, q, ℓ.



(P1) **Problema 1**

$$V_A \cdot 5a + qa \cdot 2a - \frac{12qa}{2} \cdot 3a + M = 0$$

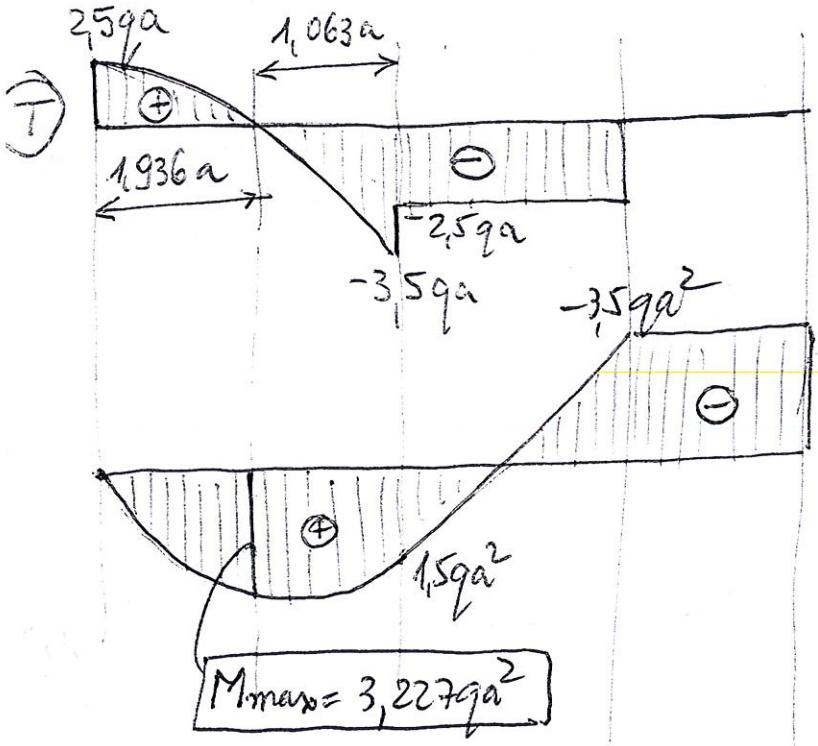
$$V_A = \frac{1}{5a}\left(18qa^2 - M - 2qa^2\right)$$

$$4q \cdot \frac{3a}{2} \cdot 2a - 3qa^2 - V_B \cdot 5a + M = 0 \Rightarrow 16qa^2 - M = 9qa^2 + M$$

$$\boxed{M = 3{,}5qa^2}$$

$$V_B = \frac{1}{5a}\left(M - 3qa^2 + 12qa^2\right)$$

$$\boxed{V_A = V_B = 2{,}5qa}$$



$$T(x) = 2{,}5qa - \frac{q_x \cdot x}{2} \Rightarrow$$

$$\frac{q_x}{4q} = \frac{x}{3a} \Rightarrow q_x = \frac{4qx}{3a}$$

$$\Rightarrow T(x) = 2{,}5qa - \frac{4qx^2}{6a} = 0$$

$$\Rightarrow 4qx^2 = 15qa^2 \Rightarrow \boxed{x = 1{,}936a}$$

$$M(x) = 2{,}5qax - \frac{2}{3}\frac{q}{a} \cdot \frac{x^3}{3}$$

$$\boxed{M_{max} = 3{,}227qa^2}$$

$$I_y = \frac{6t \cdot (10t)^3}{12} - 2\,\frac{4t(3t)^3}{12} - 2 \cdot 12t^2(2{,}5t)^2 = 332t^4$$

$$W_y = \frac{332t^4}{5t} = 66{,}4t^3$$

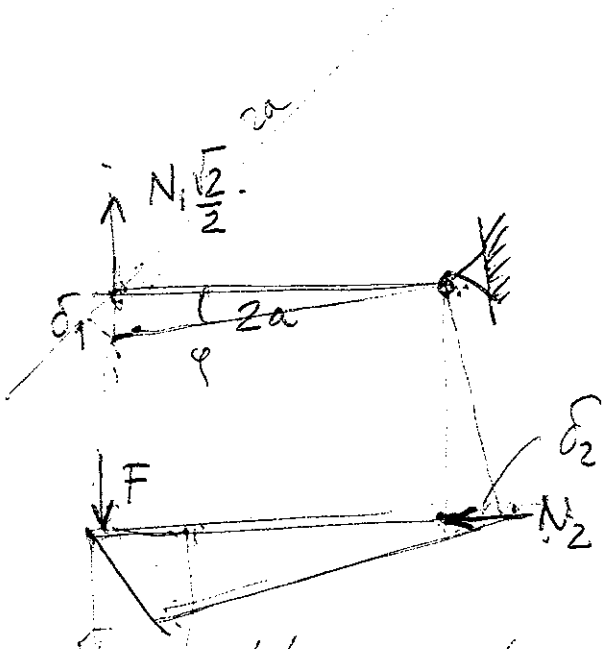
$$\sigma_{1,2} = -\frac{51{,}5}{2} \pm \frac{1}{2}\sqrt{(-51{,}5)^2 + 4 \cdot 9{,}2^2} = \begin{cases} \sigma_1 = 1{,}591 \\ \sigma_2 = -53{,}09 \text{ MPa} \end{cases}$$

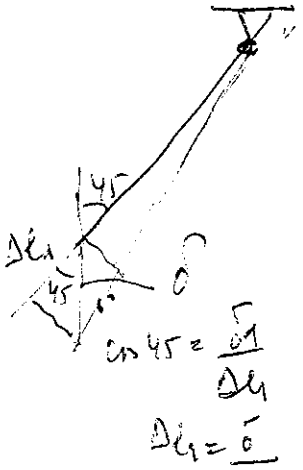
$$\sigma_{max} = \frac{3{,}5qa^2}{66{,}4t^3} \le \sigma_a \Rightarrow t \ge \sqrt[3]{\frac{3{,}5 \cdot 10 \cdot 300^2}{66{,}4 \cdot 150}} = \boxed{6{,}81 \text{ mm}}$$

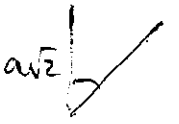
$$\sigma_K = \frac{1{,}5qa^2}{332t^4} \cdot z_K = \frac{1{,}5 \cdot 10 \cdot 300^2}{332 \cdot 6{,}81^4}\left(-4 \cdot 6{,}81\right) = -51{,}5 \text{ MPa}$$

$$S_K^* = 6t \cdot t \cdot 4{,}5t = 27t^3$$

$$\tau_K = \frac{3{,}5qa \cdot 27t^3}{2t \cdot 332t^4} = \frac{3{,}5 \cdot 10 \cdot 300 \cdot 27}{2 \cdot 332 \cdot 6{,}81^2} = \boxed{9{,}2 \text{ MP}}$$







$$N_1\frac{\sqrt{2}}{2}\cdot 2a + N_2\cdot 1{,}5a - 2Fa = 0$$

$$\boxed{N_1\sqrt{2} + 1{,}5N_2 = 2F}\ \checkmark$$

$$\frac{\delta_1}{\delta_2} = \frac{2}{1{,}5} = 1{,}33$$

$$\delta_1\frac{\sqrt{2}}{2} = \frac{N_1\cdot a}{E_{OL}\cdot A} \Rightarrow \delta_1 = \frac{N_1 a}{\sqrt{2}E_{OL}\cdot A}$$

$$\Rightarrow \frac{N_1 a}{\sqrt{2}E_{OL}\cdot A} = -1{,}33\cdot\frac{N_2\cdot 1{,}5a}{\frac{5}{3}E_{OL}A}$$

$$\frac{N_1}{\sqrt{2}} = -\frac{6\sqrt{2}}{5}N_2 = -1{,}2\sqrt{2}N_2$$

$$\delta_2 = -\frac{N_{OL}\cdot 1{,}5a}{E_{OL}A_{OL}} = -\frac{N_{AL}\cdot 1{,}5a}{E_{AL}\cdot 2A} = \frac{-N_2\cdot 1{,}5a}{E_{OL}A + E_{AL}\cdot 2A}$$

$$N_2 = N_{OL} + N_{AL}$$

$$\frac{E_{OL}A_{OL}}{E_{AL}A_{AL}} = \frac{3E_{AL}\cdot A}{E_{AL}\cdot 2A} = 1{,}5$$

$$\Rightarrow N_{OL} = \frac{N_2}{1+\frac{1}{1{,}5}} = 0{,}6N_2$$

$$N_{AL} = \frac{N_2}{1+1{,}5} = 0{,}4N_2$$

$$N_{OL} = \frac{E_{OL}A_{OL}\cdot N_2}{E_{OL}A_{OL} + E_{AL}\cdot 2A}$$

$$N_{AL} = \frac{E_{AL}\cdot A_{AL}\cdot N_2}{E_{OL}\cdot A + E_{AL}\cdot 2A}$$

$$\begin{cases} N_1\sqrt{2} + 1{,}5N_2 = 2F \\ N_1\sqrt{2} + 1{,}2N_2 = 0 \end{cases}$$

$$0{,}3N_2 = 2F \Rightarrow N_2 =$$

$$\delta_F = \sqrt{\delta_{v_F}^2 + \delta_{H_F}^2} = \sqrt{\delta_1^2 + \delta_2^2}$$

$$\delta = \sqrt{0{,}715^2 + 0{,}536^2} = 0{,}893\ mm$$

$$N_1\sqrt{2} + 1{,}5\,N_2 = 2F \quad \checkmark$$

$$\delta_1 = \frac{N_1 a}{E_{OL} A \sqrt{2}} \quad \checkmark \qquad \delta_1 = \frac{0{,}404\,F a \cdot 4}{E_{OL} \cdot A_{OL} \cdot \sqrt{2}} = 0{,}715\ mm$$

$$\delta_2 = -\frac{N_2 \cdot 1{,}5a}{\frac{5}{3} E_{OL} A}$$

$$\frac{\delta_1}{\delta_2} = \frac{2}{1{,}5} \Rightarrow \delta_2 = \frac{0{,}952 \cdot 30.000 \cdot 1{,}5\sqrt{2}}{21 \cdot 10^4 \cdot 1{,}66 \cdot 114{,}2}$$

$$\Rightarrow \frac{N_1 a \sqrt{2} \cdot 1{,}5}{E_{OL} A \sqrt{2}} = \frac{(-N_2 \cdot 1{,}5a) \cdot 3}{5 E_{OL} A}$$

$$\delta_2 = 0{,}536\ mm$$

$$\Rightarrow N_1 \cdot 1{,}5 = -\sqrt{2}\, N_2 \cdot 0{,}9$$

$$\Rightarrow \begin{cases} N_1 \cdot 1{,}5 + 1{,}341\, N_2 = 0 \ / -\sqrt{2} \ / -1{,}5 \\ N_1\sqrt{2} + 1{,}5\, N_2 = 2F \ / 1{,}5 \ / 1{,}341 \end{cases} \Rightarrow$$

$$0{,}0707\, N_1 = 2{,}682$$

$$\Rightarrow N_1 =$$

$$N_{OL} = N_2 \cdot \frac{E_{OL} A_{OL}}{\frac{5}{3} E_{OL} A_{OL}} = 0{,}6\, N_2 \quad \checkmark$$

$$N_{AL} = 0{,}4\, N_2 \quad \checkmark$$

$$\frac{\frac{4\alpha N_1}{E_{OL} A \sqrt{2}}}{\frac{-N_2 \cdot 1{,}5\alpha}{\frac{5}{3} E_{OL} A}} = \frac{2}{1{,}5} \Rightarrow \frac{10}{4{,}5\sqrt{2}} N_1 = \frac{2}{1{,}5} N_2$$

$$N_1 = \frac{4{,}5\sqrt{2}}{15} N_2$$

$$\sigma_{OL}^{max} = \frac{0{,}571\,F}{A} \le \sigma_a^{OL}$$

$$\Rightarrow A_{OL} = \frac{0{,}571 \cdot 30 \cdot 10^3}{150}$$

$$A_{OL} = 114{,}2\ mm^2$$

$$N_1^{OL} = 0{,}404\,F$$

$$N_{OL}^2 = 0{,}571\,F$$

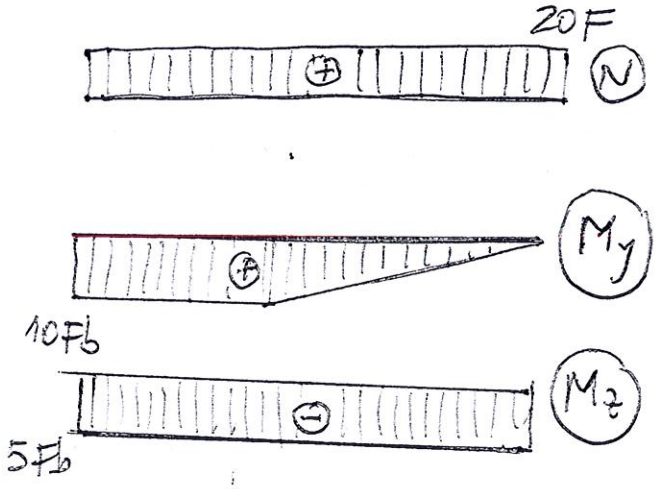
$$N_{AL}^2 = 0{,}381\,F$$

$$\sigma_{AL}^2 = \frac{0{,}381\,F}{2A} \le \sigma_{a_{AL}} \Rightarrow \left(\frac{4{,}5 \cdot 2}{15} + 1{,}5\right) N_2 = 2F \Rightarrow$$

$$\Rightarrow A_{AL} = 57{,}12\ mm^2$$

$$2{,}1\, N_2 = 2F \Rightarrow \boxed{N_2 = 0{,}952\,F}$$

Problema 3



S.P. este în capătul din stânga al barei

$$\sigma = \frac{N}{A}\left(1 + \frac{y y_0}{i_z^2} + \frac{z z_0}{i_y^2}\right)$$

$$I_y = \frac{b(2b)^3}{12} = \frac{2b^4}{3} \Rightarrow i_y^2 = \frac{2b^4}{6b^2} = \frac{b^2}{3}$$

$$I_z = \frac{2b \cdot b^3}{12} = \frac{b^4}{6} \Rightarrow i_z^2 = \frac{b^4}{12b^2} = \frac{b^2}{12}$$

$$y_0 = \frac{b}{4} \quad z_0 = \frac{b}{2}$$

$$\Rightarrow \sigma = \frac{20F}{2b^2}\left(1 + \frac{z \cdot 12}{4 \cdot b} + \frac{y \cdot 3}{2b}\right) \Rightarrow$$

$$\sigma = \frac{20F}{2b^2} + \frac{10Fb \cdot 3}{2b^4} z + \frac{5Fb \cdot b}{b^4} y$$

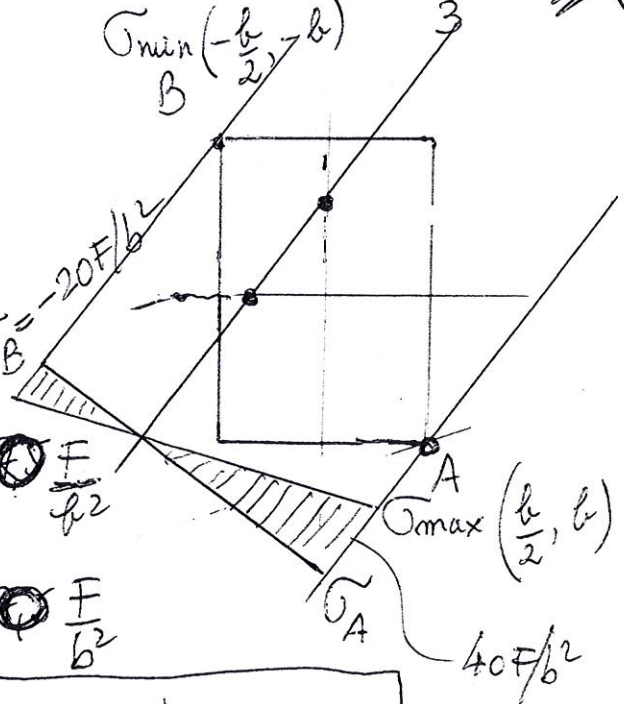
$$\sigma = \frac{20F}{2b^2}\left(1 + \frac{3z}{2b} + \frac{y}{b}\right)$$

$$1{,}5z + 3y + b = 0$$

$y = 0 \Rightarrow z = -\frac{b}{1{,}5} = -\frac{2b}{3}$

$z = 0 \Rightarrow y = -\frac{b}{3}$

$\Rightarrow$ axa neutră : $3y + 1{,}5z + b = 0$

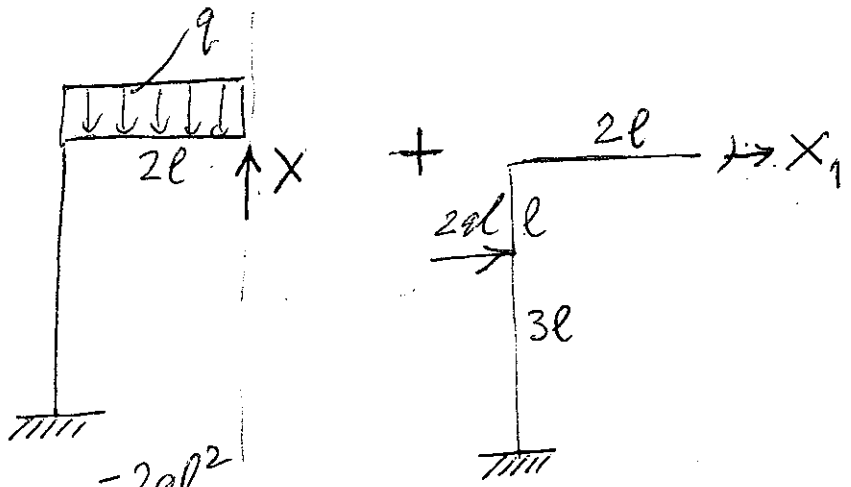


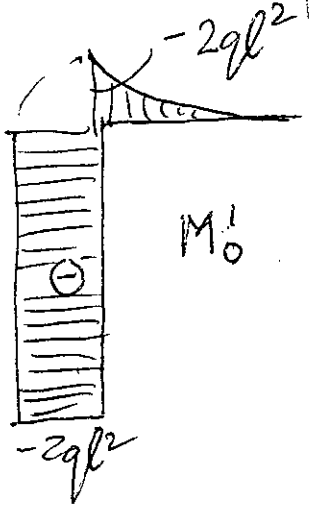
$$\sigma_{max} = \frac{10F}{b^2}\left(1 + \frac{3}{b} \cdot b + \frac{1{,}5 \cdot b}{2b}\right) = 40\frac{F}{b^2}$$

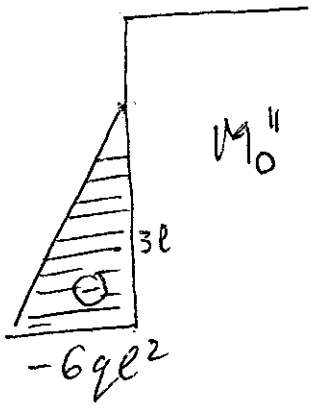
$$\sigma_{min} = \frac{10F}{b^2}\left(1 - \frac{3}{b} \cdot b - \frac{1{,}5b}{2b}\right) = -20\frac{F}{b^2}$$

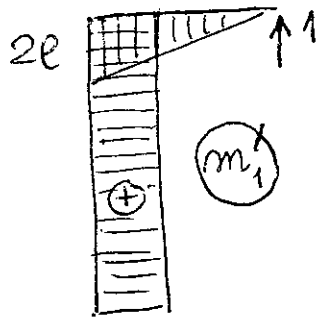
$$\left|\frac{\sigma_{max}}{\sigma_{min}}\right| = 2$$

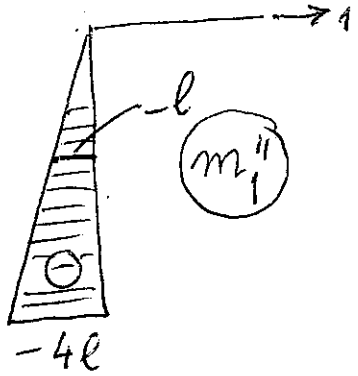
$$\text{Defl} : \frac{20F \cdot 18b}{2b^2 E} = \frac{180F}{1{,}E}$$

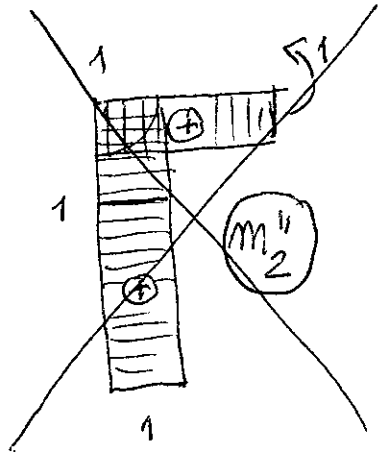












$$Ei\,\delta_{10} = \frac{2\ell}{6}\left[(-2q\ell^2 + q\ell^2)\,2\ell - 4q\ell^3\right] = -2q\ell^4 - 4q\ell^4$$

$$Ei\,\delta_{11} = 2\ell\cdot\frac{2\ell}{2}\cdot\frac{2}{3}\cdot 2\ell + 2\ell\cdot 4\ell\cdot 2\ell$$

$$Ei\,\delta_{11} = \frac{56\ell^3}{3} \Rightarrow \boxed{X = 0{,}964\,q\ell}$$

$$\boxed{X = \frac{18\cdot 3}{56}q\ell =}$$

$$\boxed{X = 0{,}964\,q\ell}$$

~~$\delta_{10} + X_1\delta_{11} + X_2\delta_{12} = 0$~~

$$Ei\,\delta_{10} = \frac{3\ell}{6}\left[(-6q\ell^2)(-5\ell) + 24\cdot q\ell^3\right] = 27\,q\ell^4$$

~~$Ei\,\delta_{20} = -6q\ell^2\cdot\frac{3\ell}{2}\cdot 1 = -9q\ell^3$~~

$$\boxed{X_1 = -1{,}26\,q\ell}$$

$$Ei\,\delta_{11} = 4\ell\cdot\frac{4\ell}{2}\cdot\frac{2}{3}\cdot 4\ell = \frac{64\ell^3}{3}$$

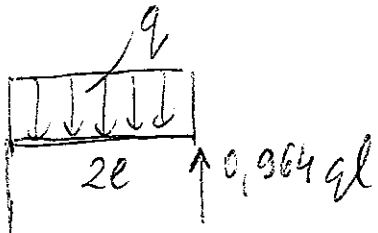
~~$Ei\,\delta_{22} = 2\ell + 4\ell = 6\ell$~~

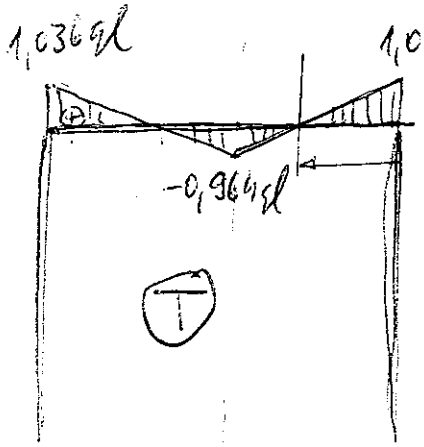
~~$Ei\,\delta_{12} = -4\ell\cdot\frac{4\ell}{2}\cdot 1 = -8\ell^2$~~

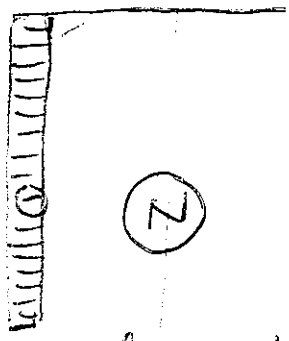
$$\begin{cases} \frac{64\ell^3}{3}X_1 - 8\ell^2 X_2 = -27q\ell^4 \\ -8\ell^2 X_1 + 6\ell X_2 = 9q\ell^3 \end{cases}$$

$$\Rightarrow \begin{cases} 64\ell X_1 - 24 X_2 = -81q\ell^2 \\ -8\ell X_1 + 6X_2 = 9q\ell^2 / 4 / 8 \end{cases}$$

~~$\boxed{X_2 = -0{,}375\,q\ell}$~~



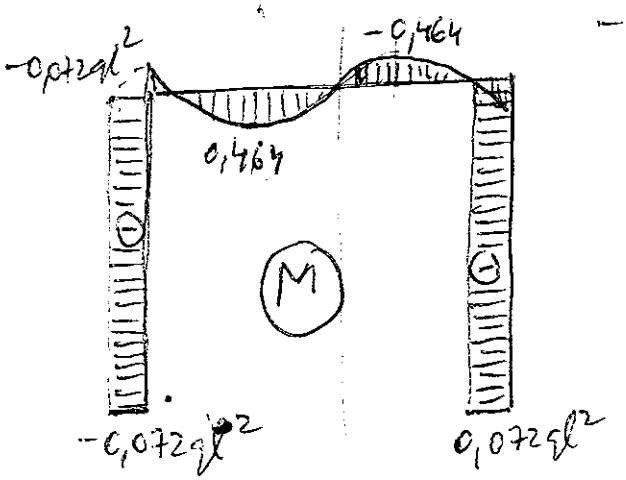




$M = 0,072 q\ell^2$ $V = 1,036\ q\ell$

$M = -2q\ell^2 + 0,964 q\ell \cdot 2\ell =$

AS



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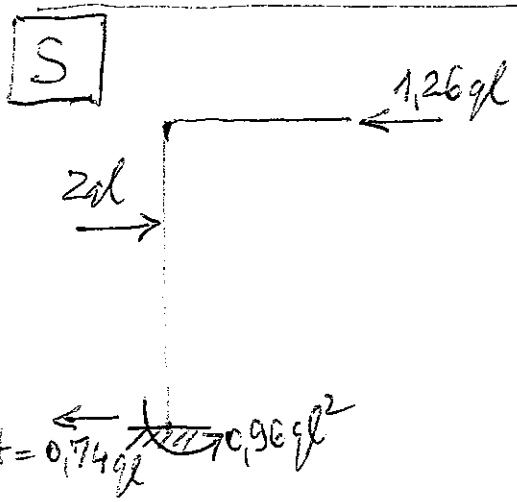

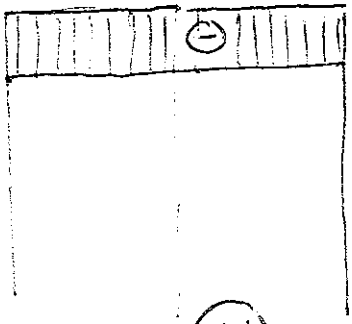

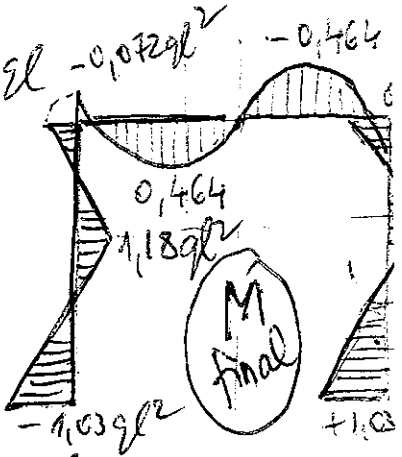

$H = 0,74\ q\ell$ $0,96\ q\ell^2$

$M = -6q\ell^2 + 1,26 q\ell(-4\ell) =$